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THE DIAMETERS OF THE STARS

BY

J. B. CANNON
III

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The question of the diameters of the stars is one which can be answered only by means of assumptions which are more or less plausible, and the results thereby arrived at can only be looked upon as being, to a large extent, very uncertain.

In order to be able to say anything about the diameter of one of the stars it is necessary to have some knowledge of three things:—(*a*) parallax, (*b*) magnitudes and (*c*) surface intensity;—to say nothing of absorption in space. From the parallax we get the distance of the star from us, distance and magnitude give luminosity, and luminosity and brightness give the surface area of the star. The sun is taken as unit in each case. Thus, having the surface of the star in terms of the sun's surface, we have at once the ratio of the radii of the star and the sun.

Now in three cases, viz.:—Algol, β Aurigæ and κ Herculis.

which have been attacked spectroscopically, the radii, masses and densities have been computed from the data thus obtained. From these results we obtain the surfaces of the stars and hence, having the luminosity, we arrive at a probable value for the intrinsic brightness or surface intensity. An example may, perhaps, be the best way to make clear the method of arriving at the luminosity of any star.

α Orionis has a parallax of $0''.03$.

$$\begin{aligned}\text{Its distance from the earth therefore} &= \frac{3.26}{.03} \text{ light years,} \\ &= 109.00 \text{ light years.}\end{aligned}$$

The magnitude of α Orionis is 0.91.

Now we know that the light-ratio between successive magnitudes is $\sqrt[4]{100}$, or the number whose logarithm is .4.

We have, therefore,

$$\log b_n - \log b_m = -\frac{4}{10}(n - m),$$

where b_n and b_m are the brightnesses, respectively, of stars of the n th and m th magnitudes. Taking a tenth magnitude star as the unit, we get

$$\begin{aligned}\log b_n &= -\frac{4}{10}(n - 10), \\ &= 4 - 0.4n.\end{aligned}$$

Using this formula, we get for the light of α Orionis

$$\begin{aligned}\log \text{of brightness of } \alpha \text{ Orionis} &= 4 - 0.4(0.91) \\ &= 3.636.\end{aligned}$$

Now the luminosity is equal to the light multiplied by the distance squared.

Hence, in terms of the sun's luminosity, we have

$$\begin{aligned}\text{Luminosity of } \alpha \text{ Orionis} &= \frac{\log^{-1} 3.636 \times [109 \times 365 \times 24 \times 60]^2}{\log^{-1} 14.52^* \times [8]^2}, \\ &= 670 \odot\end{aligned}$$

where $\odot$ denotes the luminosity of the sun.

* These are obtained from taking -26.3 for the sun's magnitude and 8 light-minutes for its distance from the earth.

The luminosities used in the table which follows were taken from Kapteyn and Weersma's publication mentioned hereafter. They obtained them from the formula

$$\log L = 0.200 - 0.4 m - 2 \log \pi.$$

This is the same as that used above as will be seen from the formula,

$$\log L = \log \left(\frac{\text{brightness of star}}{\text{brightness of sun}} \right) \times \left(\frac{\text{distance of star}}{\text{distance of sun}} \right)^2,$$

$$\text{or } \log L = (4 - 0.4 m) - (4 - 0.4 m \text{ of sun}) \times \left(\frac{\text{distance of star}}{\text{distance of sun}} \right)^2.$$

Now the sun's magnitude was taken as 5.5 at a distance at which the parallax is 1".

Hence the formula becomes

$$\begin{aligned} \log L &= (4 - 0.4 m) - [4 - 0.4 (5.5)] + \log \left[\frac{\frac{3.26}{\pi''}}{\frac{3.26}{1''}} \right]^2 \\ &= 0.22 - 0.4 m - 2 \log \pi. \end{aligned}$$

Having then a star of any type whose surface intensity is known and whose radius is known we are in a position to speculate as to the radii of other stars of the same type. For other types we have to assume some relation between the surface intensity of the various types and that of the sun. That is what has been done here.

The star which was taken as a standard to work from is β Aurigæ, discussed by Stebbins in the *Astrophysical Journal*, Vol. XXXII., p. 185, October, 1910. He has classed it as belonging to *Ap*, and has obtained the following results from the assumption of a parallax of 0".03:—

Radius of each component	-	-	-	2.58 ☉.
Mass of 1st	-	-	-	2.38 ☉.
Mass of 2nd	-	-	-	2.34 ☉.
Density of each	-	-	-	0.14 ☉.
Total light	-	-	-	greater than 80 ☉.
Surface intensity	-	-	-	greater than 12 ☉.

In the following, the classes have been grouped very liberally, *A* and *B* being taken together, as having the same surface intensity, viz., that given by Stebbins above for β Aurigæ, namely, 12 $\odot$; *F* has been taken as 3 $\odot$; *G* equal to $\odot$ and *K* and *M* one-half $\odot$.

The luminosities were taken from the *Publications* of the Astronomical Laboratory at Groningen, No. 24, (being a list of parallax determinations compiled by Professor Kapteyn and Dr. Weersma). In three cases, Algol and α and β Orionis, the luminosities were not given by them, these stars having parallaxes 0".03 or lower, from which Kapteyn and Weersma consider no reliable values can be obtained. In these three cases the luminosities have been computed from the magnitude and parallaxes and expressed in terms of the sun.

CLASS I (TYPES *A* AND *B*) SURFACE INTENSITY = 12 $\odot$

Star	Luminosity	Radius
α Aquilæ (Altair)	12.3	1
α Canis Majoris (Sirius)	48.1	2
α Crucis	(122.0	3)
α Eridani (Achernar)	76.6	2.4)
α Leonis (Regulus)	350	5.4
β Leonis (Denebola)	423	6
α Lyre (Vega)	12.2	1
β Orionis (Rigel)	158	3.6
	20614	41.5
ζ Ursæ Maj. (Mizar)	(159	3.6)
β Persei (Algol)	(39	1.8)
	238	4.5

CLASS II. (TYPE *F*) SURFACE INTENSITY = 3 $\odot$

Star	Luminosity	Radius
α Canis Minoris (Procyon)	9.68	1.8
α Ursæ Min. (Polaris)	102.	5.9

CLASS III. (TYPE *G*) SURFACE INTENSITY = $\odot$

Star	Luminosity	Radius
α Aurigæ (Capella)	300	17.3
α Centauri	2.03	1.4
β Geminorum (Pollux)	127	11.2

CLASS IV. (TYPES *K* AND *M*) SURFACE INTENSITY = $\frac{1}{2} \odot$

Star	Luminosity	Radius
α Bootis (Arcturus)	230	21.4
α Tauri (Aldebaran)	224	15.0
α Orionis (Betelgeux)	524.3	32.4

A very interesting article appears in *Comptes Rendus* for January 9, 1911, in which M. Chas. Nordmann computed the values of the ratio $\frac{R_*}{R_\odot}$ from the following formula

$$\log \frac{R_*}{R_\odot} = \log \frac{d_*}{d_\odot} - \left[\frac{1}{5} (G_* - G_\odot) + \frac{1}{2} \log \frac{E_*}{E_\odot} \right],$$

where R_* and $R_\odot$ are the semi-diameters of a star and the sun, G_* and $G_\odot$ their magnitudes, d_* and $d_\odot$ their distances from the earth and E_* and $E_\odot$ their intrinsic brightness. He finds the following values:—

Star	$\frac{R_*}{R_\odot}$	Value given above
Sirius	1.13	2
Procyon	1.35	1.8
Aldebaran	13.50	15.0
Capella	8.26	17.3
Vega	1.57	3.0
γ Cygni	0.92	...
α Persei	1.83	...
Polaris	1.93	5.9
δ Andromedæ	13.00	...
δ Persei	1.29	4.5

Considering the assumptions that were employed, the agreement between the results above must be regarded as very fair.

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